

Bounding CPT- and Lorentz-Violating Parameters in a Standard-Model Extension^a

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A general theoretical framework that incorporates possible CPT and Lorentz violation in an extension of the standard model and in quantum electrodynamics has been developed over the last decade. The framework originates in the idea that CPT and Lorentz symmetry could be broken spontaneously in a more fundamental theory such as string theory. These symmetry violations are described in the standard-model extension by small terms in the Lagrangian. Various experiments can bound these quantities. They include Penning-trap experiments with electrons and positrons, Penning-trap experiments with protons and antiprotons, and possible experiments with hydrogen and antihydrogen in traps or beams. I will review aspects of the theory, outline estimated bounds attainable in specific experiments, and present known bounds from completed experiments.

1 Introduction

Numerous experiments have searched for violations of CPT and Lorentz symmetry. These include tests with electrons and positrons¹ and kaons.² Up until recently, no theoretical framework incorporating such violations has been available to provide suggestions for promising signals from the theoretical standpoint. This proceedings will provide an overview of work done in collaboration with Alan Kostelecký and Robert Bluhm aimed at identifying interesting experimental tests in particle-trap systems from theoretical considerations. The context is an extension of the standard model of particle physics and electrodynamics³ that includes the possibility of small violations of CPT and Lorentz symmetry while maintaining the important conventional features of the standard model. Several experiments searching for the miniscule effects in this theoretical context have been performed, and are briefly mentioned. The framework has recently been used to investigate signals of Lorentz and CPT violation in atomic clock-comparison experiments⁴ and the muon system.⁵ References on the standard-model extension and a broader overview of experimental tests may be found in a recent review.⁶

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2 Standard-Model Extension

The origin for the extension of the $SU(3) \times SU(2) \times U(1)$ standard model and quantum electrodynamics is the concept of spontaneous CPT and Lorentz breaking in a fundamental framework such as string theory.^{7,8} This framework lies within the context of conventional quantum field theory and appears to preserve a number of important conventional features of the standard model such as gauge invariance, power-counting renormalizability, and microcausality. Possible violations of CPT and Lorentz symmetry enter into the theory through couplings that could potentially give experimental signals. Experiments with the potential to place bounds on the couplings include investigations with trapped hydrogen (H) or antihydrogen (\bar{H}), and with Penning traps to be discussed here.

In this context, the Dirac equation obeyed by a four-component spinor field ψ describing a particle with charge q and mass m has the form^{3,9}

$$\begin{aligned} (i\gamma^\mu D_\mu - m - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + i c_{\mu\nu} \gamma^\mu D^\nu + i d_{\mu\nu} \gamma_5 \gamma^\mu D^\nu \\ + i e_\mu D^\mu - f_\mu \gamma_5 D^\mu + \frac{1}{2} i g_{\mu\nu\lambda} \sigma^{\mu\nu} D^\lambda) \psi = 0 \quad .(1) \end{aligned}$$

Here, $iD_\mu \equiv i\partial_\mu - qA_\mu$ and A^μ is the electromagnetic potential. The equation is given in terms of a set of effective coupling constants, a_μ , b_μ , $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$, e_μ , f_μ , and $g_{\mu\nu\lambda}$. Of these, $H_{\mu\nu}$ is antisymmetric, $c_{\mu\nu}$, $d_{\mu\nu}$ are traceless, and all are real. We note that the terms involving a_μ , b_μ , e_μ , f_μ , $g_{\mu\nu\lambda}$ break CPT while those involving $H_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ preserve it. All eight terms are observer Lorentz covariant, but break particle Lorentz symmetry. It is possible to eliminate all the e_μ and f_μ terms and some of the $g_{\mu\nu\lambda}$ components to first order by a field redefinition. It will suffice here to set e_μ , f_μ , $g_{\mu\nu\lambda}$ equal to zero.³ Superscripts on the couplings will be used where needed to distinguish, for example, electron-positron sector couplings from proton-antiproton sector couplings.

The CPT- and Lorentz-violating couplings must all be minuscule since no breaking of these symmetries has been observed to date. A natural suppression scale for these quantities is the ratio of a light scale to a scale of order of the Planck mass arising in a more fundamental model.

3 Symmetry Tests with Antihydrogen and Hydrogen

We have investigated tests of CPT symmetry in H and \bar{H} using both free and trapped atoms. For the free case,¹⁰ the possible signals affecting the 1S-2S transitions in the context of the standard-model extension are suppressed by at least two factors of the fine-structure constant.

3.1 The 1S to 2S transitions in the trapped case

Experiments for the creation of $\overline{\text{H}}$ ^{11,12} propose to conduct spectroscopic measurements on H or $\overline{\text{H}}$ held within a magnetic trap with an axial bias magnetic field. The Ioffe-Pritchard trap¹³ confines states where the magnetic dipole moment is opposite in direction to the magnetic field. The magnetic bias field B splits the 1S and 2S levels into four hyperfine Zeeman levels, which we denote in order of increasing energy, by $|a\rangle_n$, $|b\rangle_n$, $|c\rangle_n$, $|d\rangle_n$, with principal quantum number $n = 1$ or 2 , for both H and $\overline{\text{H}}$.

Only the $|c\rangle$ and $|d\rangle$ states are trapped, so it is relevant to consider transitions involving these states. For small values of the magnetic field, transitions between the $|d\rangle_1$ and $|d\rangle_2$ states are field independent. It would therefore seem experimentally advantageous to compare the frequency ν_d^H for the 1S-2S transition $|d\rangle_1 \rightarrow |d\rangle_2$ in H with the corresponding frequency $\nu_d^{\overline{H}}$ in $\overline{\text{H}}$, because the magnetic-field stability and inhomogeneity would be less critical. However, there are again no unsuppressed frequency shifts in this transition. The same holds for $\overline{\text{H}}$. So, for this transition, we find $\delta\nu_d^H = \delta\nu_d^{\overline{H}} \simeq 0$ at leading order.

Another possibility would be to consider the 1S-2S transition between the mixed-spin states $|c\rangle_1$ and $|c\rangle_2$ in H and $\overline{\text{H}}$. We find that an unsuppressed frequency shift occurs in 1S-2S transitions between the $|c\rangle_1$ and $|c\rangle_2$ states because the n dependence in the hyperfine splitting produces a spin-mixing difference between the 1S and 2S levels. The frequency shift due to the CPT- and Lorentz-violating terms is field dependent with a maximum at about $B \simeq 0.01$ T. However, an experimental constraint is also relevant here. The 1S-2S transition $|c\rangle_1 \rightarrow |c\rangle_2$ in H and $\overline{\text{H}}$ is field dependent, and so the inhomogeneous trapping fields could lead to significant Zeeman broadening in the line.

3.2 Hyperfine transitions in the 1S level

We next consider frequency measurements of transitions in the hyperfine Zeeman sublevels. The CPT- and Lorentz-violating couplings in the standard-model extension give rise to field-dependent energy shifts of the $|a\rangle$ and $|c\rangle$ hyperfine levels and field-independent shifts of the $|b\rangle$ and $|d\rangle$ hyperfine levels in the 1S ground state of H.

A favorable transition is the $|d\rangle_1 \rightarrow |c\rangle_1$ line. At a magnetic field of about 0.65 T, the frequency shifts in H and $\overline{\text{H}}$ have opposite signs and are unsuppressed. The difference in the frequencies $\nu_{c \rightarrow d}^H$ and $\nu_{c \rightarrow d}^{\overline{H}}$, calculated to leading order, is $\Delta\nu_{c \rightarrow d} \equiv \nu_{c \rightarrow d}^H - \nu_{c \rightarrow d}^{\overline{H}} \approx -2b_3^p/\pi$. It isolates the CPT-violating coupling b_3^p for the proton and is therefore of interest as a clean test of CPT within the standard-model extension.

An appropriate figure of merit $r_{rf,c\rightarrow d}^H$ can be defined¹⁰ and it is found that

$$r_{rf,c\rightarrow d}^H \approx \frac{2\pi|\Delta\nu_{c\rightarrow d}|}{m_H} \quad , \quad (2)$$

where m_H is the atomic mass of H. Assuming a frequency resolution of about 1 mHz would be possible, we estimate an upper bound of $r_{rf,c\rightarrow d}^H \lesssim 5 \times 10^{-27}$. The corresponding limit on the CPT- and Lorentz-violating coupling b_3^p would be $|b_3^p| \lesssim 10^{-18}$ eV. This is more than four orders of magnitude better than bounds attainable from 1S-2S transitions.

It is worth noting that the frequency resolution of high-precision clock-comparison experiments, which can also constrain Lorentz violation, lies below 1 μ Hz. The complex nuclei involved, however, make the theoretical interpretation of these experiments more difficult.⁴

A feature of the $|d\rangle_1 \rightarrow |c\rangle_1$ transition is time-dependence of the shift due to the changing angle between the b_j^p vector and the magnetic field of the trap. Relevant figures of merit for diurnal-variation signals of this type, as well as other instantaneous comparisons for H and \bar{H} are discussed elsewhere.¹⁰

4 Symmetry Tests with Penning Traps

The behavior of an electron or other charged particle in a Penning trap is characterized by several frequencies including the cyclotron and anomaly frequencies, which can be measured to precisions better than one part in 10^8 . For an electron or positron, the leading-order shifts of these frequencies in the framework of the standard-model extension are

$$\omega_c^{e-} \approx \omega_c^{e+} \approx (1 - c_{00}^e - c_{11}^e - c_{22}^e)\omega_c \quad , \quad (3)$$

$$\omega_a^{e\mp} \approx \omega_a \mp 2b_3^e + 2d_{30}^e m_e + 2H_{12}^e \quad . \quad (4)$$

Here, ω_c and ω_a denote the unperturbed frequencies, while $\omega_c^{e\mp}$ and $\omega_a^{e\mp}$ represent the frequencies including the corrections. These two expressions contain the dominant corrections in the CPT- and Lorentz-breaking quantities, and in the magnetic field.

4.1 Anomalous Magnetic Moments

Using Eqs. (3) and (4), the electron-positron differences for the cyclotron and anomaly frequencies are

$$\Delta\omega_c^e \equiv \omega_c^{e-} - \omega_c^{e+} \approx 0 \quad , \quad \Delta\omega_a^e \equiv \omega_a^{e-} - \omega_a^{e+} \approx -4b_3^e \quad . \quad (5)$$

It follows that the dominant signal for CPT breaking in Penning-trap $g - 2$ experiments is a difference between the electron and positron anomaly frequencies. No leading-order contributions appear from CPT preserving but Lorentz breaking terms. An appropriate figure of merit can be introduced in a general context as the ratio of a CPT-violating electron-positron energy-level difference and the basic energy scale ¹⁴

$$r_{\omega_a}^e \equiv \frac{|\mathcal{E}_{n,s}^{e-} - \mathcal{E}_{n,-s}^{e+}|}{\mathcal{E}_{n,s}^{e-}} . \quad (6)$$

Here, $\mathcal{E}_{n,s}^{e-}$ and $\mathcal{E}_{n,s}^{e+}$ are energy eigenvalues for the full Penning-trap hamiltonians, with quantum numbers $n = 0, 1, 2, \dots$ and spin $s = \pm 1$. Within the present nonrelativistic framework $\mathcal{E}_{n,s}^{e-} \rightarrow m_e$ and the energy difference in the numerator becomes half the difference between the two measured anomaly frequencies, $\Delta\omega_a^e/2 \approx -2b_3^e$. Thus, in the present context (6) reduces to

$$r_{\omega_a}^e \approx \frac{|\Delta\omega_a^e|}{2m_e} \approx \frac{|2b_3^e|}{m_e} . \quad (7)$$

If the anomaly frequencies were measured to an accuracy of 2 Hz, then this experiment would place a bound of $r_{\omega_a}^e \lesssim 10^{-20}$.

It is important to note that past CPT tests ¹ have made high-precision comparisons of the g factors of electrons and positrons, to bound the conventional figure of merit

$$r_g \equiv \left| \frac{(g_- - g_+)}{g_{\text{av}}} \right| \lesssim 2 \times 10^{-12} . \quad (8)$$

Within the framework of the standard-model extension, however, CPT is broken without affecting the electron or positron gyromagnetic ratios. Thus, the theoretical value of r_g would be zero even if CPT were broken, and r_g used in these experiments is unsuitable as a CPT figure of merit in this context.

4.2 Diurnal anomaly-frequency signals

Various experiments searching for diurnal signals are possible.⁹ One option is to search for a diurnal signal in the anomaly frequency for an electron alone, or for a positron alone. In the standard-model extension, this variation would occur because the components of the couplings in Eq.(4) would change as the earth rotates. Consider

$$\Delta_{\omega_a^{e-}}^e \equiv \frac{|\mathcal{E}_{0,+1}^{e-} - \mathcal{E}_{1,-1}^{e-}|}{\mathcal{E}_{0,-1}^{e-}} . \quad (9)$$

A suitable figure of merit $r_{\omega_a^-, \text{diurnal}}^e$ may be defined as the amplitude of the diurnal variation in $\Delta_{\omega_a^e}^e$.

Data from an experiment confining a single electron in a Penning trap has recently been analyzed by Mittleman of the Dehmelt trapping group at the University of Washington in Seattle, WA. The bound produced in this experiment is on a figure of merit related to $r_{\omega_a^-, \text{diurnal}}^e$. To search for diurnal variations, the data was split up into several bins over the sidereal day according to the orientation of the experimental magnetic field. The bound obtained¹⁵ is

$$r_{\omega_a^-, \text{diurnal}}^e \leq 1.6 \times 10^{-21} \quad . \quad (10)$$

This places a new constraint on a combination of Lorentz- and CPT-violating quantities in the standard-model extension.

Another result¹⁶ has recently been obtained by Dehmelt and collaborators at the University of Washington. Using previously obtained data from g-2 experiments comparing single trapped electrons and single trapped positrons, a bound of

$$r_e < 1.2 \times 10^{-21} \quad (11)$$

was found on a figure of merit closely related to the one defined in Eq. (6).

4.3 Hydrogen Ions and Antiprotons

In Penning-trap experiments comparing protons and antiprotons,¹⁷ the main measurement uncertainty is due to the technical difficulties in precisely reversing the electrostatic potential on the electrodes when particles of opposite charge are loaded. To overcome these constraints Gabrielse and coworkers refined their experiment to simultaneously trap an antiproton and a hydrogen ion.¹⁸ The idea is that the H^- ion substitutes for the proton and corrections are made for the electrons in the ion. Since both trapped particles have the same charge, problems of reversing the electric-field polarity are eliminated. Measurements of the cyclotron frequency can be made on each particle independently, and with greater frequency than previously. The theoretical value of the difference $\Delta\omega_c^{H^-} \equiv \omega_c^{H^-} - \omega_c^{\bar{p}}$ is obtained in conventional quantum theory using established precision measurements of the electron mass and the H^- binding energy. Comparison of this theoretical value with the experimental result for $\Delta\omega_c^{H^-}$ Provides a proton-antiproton charge-to-mass test.

Employing this technique, Gabrielse's group recently obtained the result

$$r_{q/m}^p \equiv \left| \frac{[(q_p/m_p) - (q_{\bar{p}}/m_{\bar{p}})]}{(q/m)_{\text{av}}} \right| \lesssim 9 \times 10^{-11} \quad (12)$$

for the proton-antiproton charge-to-mass ratio comparison.¹⁸ Apart from this ten-fold improvement over the previous value, the experiment also placed the bound

$$r_{\omega_c}^{H^-} \lesssim 4 \times 10^{-26} \quad (13)$$

on a Lorentz-violation figure of merit⁹ within the framework of the standard-model extension.

5 Discussion

Other possibilities for detecting signals of CPT and Lorentz violation not discussed here include diurnal frequency variations and cyclotron-frequency measurements in Penning traps,⁹ and alternative transitions in H and $\bar{\text{H}}$ atoms. Furthermore, the electron-positron Penning-trap experiments could in principle be performed with protons and antiprotons. Experiments involving boosted particles, for example H or $\bar{\text{H}}$ beams,¹⁹ have an enhanced sensitivity to time-like components²⁰ like b_0^e and b_0^p . In this summary, several interesting cases of signals that might occur in Penning-trap and H or $\bar{\text{H}}$ experiments have been discussed. The theoretical investigation is based on the framework provided by the standard-model extension.³ Three experiments mentioned above have ruled out signals of Lorentz and CPT violation at current precisions, but future investigations could still find evidence of the breaking of these fundamental symmetries either through improved precisions or through as yet unperformed experiments.

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